gymnasiumliestal

Matura Examination 2016 – Mathematics (written)

Classes: 4GL and 4Wb

Remarks:	The duration of this exam is 4 hours.
	Start each question on a fresh sheet of paper!
Material:	Calculator TI-Nspire CAS in Press-To-Test-Mode.
	Fundamentum Mathematik and Physik, Orell Füssli Verlag, plus Formula Booklet in English
	- neither may contain hand-written notes

Question 1: Vector Geometry (12 points)

The plane $E : 3x + 2y + 6z - 18 = 0$ together with the lines

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 7 \end{pmatrix} + s \cdot \begin{pmatrix} -5 \\ 1 \\ -6 \end{pmatrix} \quad \text{and} \quad h: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ 2 \\ 5 \end{pmatrix} \quad \text{are given.}$$

(a) Calculate the coordinates of the intersection points of plane E with the principal axes. Also, sketch the plane E and both of the lines g and h onto the coordinate axes printed on page 2. [2]

b) Calculate, **by hand**, the coordinates of the intersection point, S, of the lines g and h. [2]

(c) Calculate the angle between the lines g and h. [1]

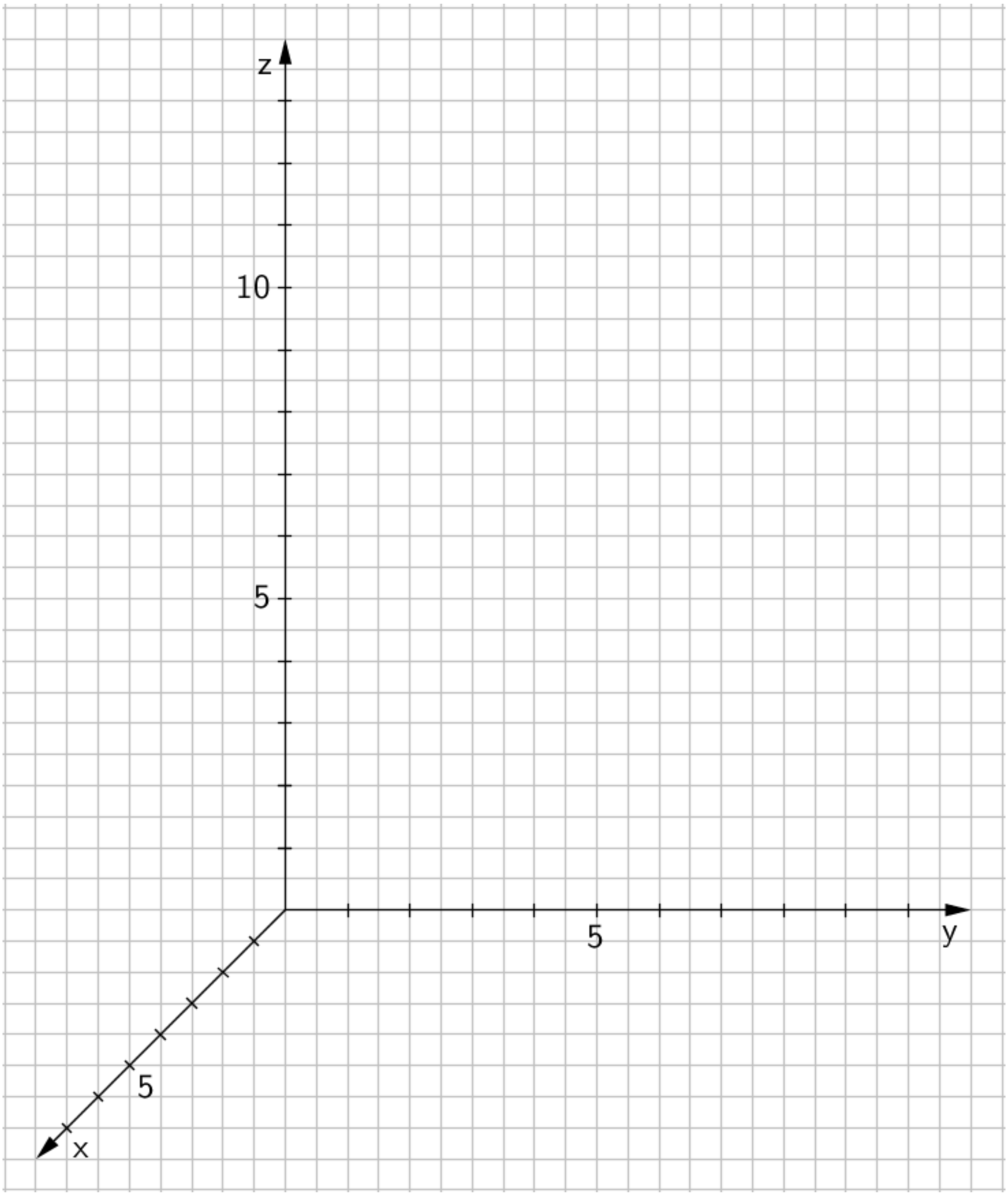
A pyramid is formed using the summit-point $S(8 \mid 7 \mid 13)$ together with the triangular base $F_S GH$, where F_S is the point in plane E which lies closest to point S, while $G(-2 \mid 9 \mid 1)$ and $H(-2 \mid 3 \mid 3)$ are the intersection points with plane E of the lines g and h, respectively.

(d) Calculate the coordinates of F_S. [2]

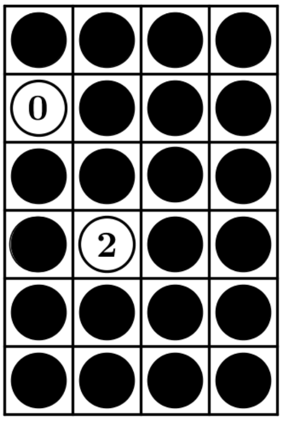
(e) Calculate the distance of the summit-point S from the base plane of this pyramid and also its volume. [2.5]

(f) A light ray beamed from the summit-point, S, strikes plane E at $P(4 \mid 3 \mid 0)$ and is then reflected in this plane. Find an equation for the line which describes the path of the reflected light ray. [2.5]

Oblique Coordinate Axes for Question 1(a)



Question 2: Probability (11 points)



Lottery scratch cards (see diagram) are put on sale.
Each card has 24 squares, each square concealing – in a random arrangement – the numbers 0, 1, 2 and 3. On each card are printed nine 0s, seven 1s, five 2s and three 3s. To play this lottery, exactly two squares of the card must be rubbed.

a) Show that the following probabilities for the events A, B and C, are:
 P(A) = 7.61%, P(B) = 61.96% and P(C) = 25.72%,
accurate to two decimal places where
A = Both rubbed squares reveal a 1. [1]
B = At least one 0 is revealed. [1]
C = Two different numbers - both bigger than 0 - are revealed. [2]

b) If neither of the numbers revealed is a 0, then the card becomes a winning card. Alastair buys and rubs ten cards. What is the probability that exactly four of these cards are winning cards? [1.5]

c) What is the smallest number of cards that Sandra must purchase in order to have at least a $\frac{2}{3}$ chance of obtaining "a pair of 3s" on *at least one* of her cards? [1.5]

A lottery scratchcard costs Fr. 2.−.
The rules concerning prizes for correctly rubbed cards are as follows:

- When at least one of the two numbers is a 0, the card has no value.
- When two different numbers greater than 0 appear, the prize is Fr. 2.-
- With two 1s, the prize is Fr. 5.- ; for two 2s the prize is Fr. 15.- and for two 3s a prize of Fr. 30.−.

d) The random variable, X, represents the net gain/loss from the viewpoint of the person who buys a lottery scratch card. Construct a table showing the different values that X can take, together with their respective probabilities. Justify, with a clear calculation, whether you would recommend participating in this lottery. [3]

e) Paula buys 24 lottery scratch cards. What overall net gain (or net loss) can she expect from this? [1]

Question 3: Calculus I (15 points)

The function $g(x) = \frac{x^3}{2} - x^2 - 2x$ is given.

Advice: *Part questions 3(a), 3(b) and 3(c) are to be solved* ***by hand*** *.*

a) Calculate the coordinates of any maximum/minimum points on the curve of function $g(x)$. [4]

b) Calculate the coordinates of any inflection points on the curve of $g(x)$. [2.5]

c) The line, ℓ, is drawn vertically through the minimum point of the graph of function $g(x)$. The bounded region, A_T, is created between the line ℓ, the curve of $g(x)$ and the x-axis, for $0 \le x \le 2$. Calculate the area of region A_T. [2]

Advice: *For the remaining parts of Question 3, the full use of your calculator is permitted.*

d) Between the parabola formed by the curve of $p(x) = -(x - 2)^2 + 3$ and the curve of $g(x)$ lie two bounded regions. Calculate a value for the ratio of the areas of these two regions. [2.5]

e) Investigate whether every polynomial of degree 3, given in the form: $f(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$ where $a \neq 0$, has exactly one point of inflexion and, if so, give its coordinates. [2]

f) Find an equation to represent the particular polynomial of degree 3 that has point symmetry about $O(0 \mid 0)$ and has a local minimum at $T(2 \mid -5)$. [2]

Question 4: Calculus II (10 points)

The function $h(x) = (2x - x^2) \cdot e^{\frac{x}{2}}$ is given, where e is Euler's number.

Advice: *The part-questions 4(a) und 4(b) are to be solved* ***by hand****.*

a) Find the zero points of function $h(x)$. [1]

b) Show that $H(x) = (-2x^2 + 12x - 24) \cdot e^{\frac{x}{2}}$ is a possible integral function for $h(x)$. [1.5]

Advice: *For the remaining parts of Question 4, the full use of your calculator is permitted. Where necessary, your answers should be expressed to an accuracy of three decimal places.*

c) A rectangle lies completely inside the 3rd quadrant of the graph, so that one corner is at $O(0|0)$, and its diagonally-opposite corner is point Q, where Q is a point on the graph of $h(x)$.

Calculate the coordinates of the point, Q , so that the rectangle has a maximum area. What is the value of this optimal area? [3.5]

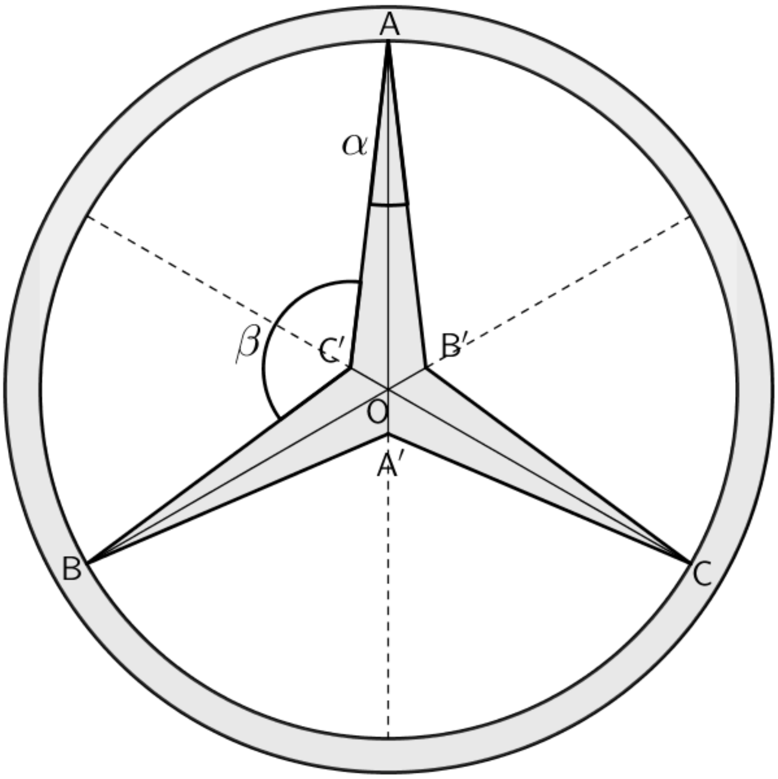
d) In the 1st quadrant, a bounded region is formed between the curve of $h(x)$ and the x-axis. This region represents a real piece of land, where a scale of one unit on the graph represents 100 metres on the land. The owner wishes that, on his death, this piece of land will be divided into two regions with equal areas for his son and daughter to inherit. The boundary between these two regions is to be line ℓ_{NP}, with $N(1|0)$ and point P lying on the curve of $h(x)$.

Find the coordinates of point P, as well as the area of land in square metres that each child would receive. [4]

Half Question 5a: Trigonometry (6 points)

Company logos are often created from simple geometric shapes. Below can be seen the logo of a popular car manufacturer.

This grey-coloured logo has been created using a **regular** 3 – pointed star together with a circular ring: the central point of both of these shapes lies at O.
The radius of the inner circle is r = 4 cm, and that of the outer circle is R = 4.4 cm.
A consequence of the above lengths is that the circular ring has an area which is twice as big as the area of the star itself.



Advice: *Your results should be rounded to 3 decimal places.*

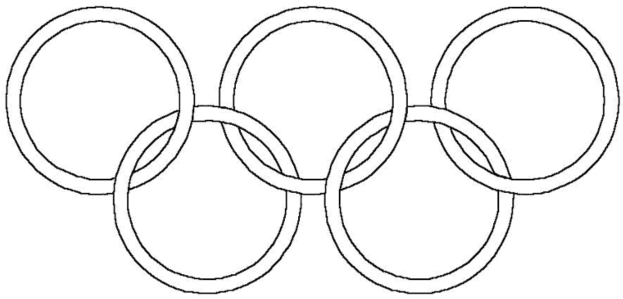
a) Calculate the area of the star shape and show that length $\overline{OB'} = 0.508$ cm. [3]

b) Calculate the length of a single side of this star (E.g. $\overline{AB'}$). [1]

c) Find the sizes of angles $\alpha = C'AB'$ and $\beta = AC'B$ [2]

Half Question 5b: Combinations & Permutations (6 points)

While looking through his colouring book, Brian finds the symbol (das Motiv) of the Olympic Games. The instructions state that each *individual* ring should be fully shaded using only one colour.



a) Brian's pencil case contains 12 pencils, each of a different colour.
In how many ways can this symbol be coloured in, if different rings may be shaded with the *same* colour? [1]

b) In a single grip, he now removes 5 pencils from the pencil case.

i. How many different groups of 5 pencils could he have chosen? [1]

ii. In how many ways can the whole symbol be coloured in using the pencils chosen, if each pencil may be used to colour one - and only one - ring? [1]

c) In how many different ways can the Olympic symbol be coloured using only the blue, the yellow and the red pencil, given that each of these pencils must be used at least once? [3]