Matura 2012 – Written exam in Mathematics

Classes: 4(B)M, 4GL, 4I, 4LW, 4LZ, 4S, 4SW, 4Wa, 4Wb

4LW bil. e.

Duration :	4 hours
Approved Materials:	Calculator and handbook. (TI-89, TI Voyage 200, TI N-spire and a non-CAS calculator) Formula Sheet (in English) and *'Fundamentum'* (in German). Dictionary (German/English) available on the front desk.
Remarks :	Each question has 12 marks. Part-marks are shown in brackets. Start each question on a fresh sheet of paper. Show all your work.

1. Vector Geometry

The three points, A(–2|–1|1), B(1|3|0) und C(5|–5|–3), are given, as well as the parametric equations of the line g: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$ and the line h: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ -6 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$.

a) Find a Cartesian equation for the plane ε which passes through points A, B, and C. Then show that the line g and the line h intersect at the point S(2|6|10). [3]

N.B.: If you were unable to find a Cartesian equation for plane ε in part a), you should continue with the plane $\varepsilon: 4x - y + 8z - 1 = 0$.

b) Calculate the coordinates of S*, image of the point S through reflection in plane ε. [3]

c) Calculate the volume of the double tetrahedron ABCSS* formed by joining two tetrahedra at their common face ABC. [3]

d) Calculate the (acute) angle α between the ridge BS and the face ABC, as well as the (acute) angle β formed between the face BCS and the face ABC. [3]

2. Calculus

Where appropriate, your results to this question should be given accurate to 4 decimal places.

Given the function: $f(x) = \frac{3}{40}x^3 - \frac{1}{5}x^2 + \frac{1}{10}x$,

a) Calculate the complete coordinates of all maximum points on the graph of f. [2]

b) Find the value(s) for x where the graph of f has gradient equal to 2. [1]

c) *To obtain full marks for this part-question, you must show a clear solution that does not use the calculator.*
Calculate all zero points of f. [2]

d) The positive x–axis together with the graph of f produce two bounded regions. By what factor is the area of the larger region *bigger* than the area of the smaller region? [2]

For the remainder of Question 2, the coefficient $\frac{3}{40}$ in function f is replaced by parameter t, where t > 0.

Thus, we have the family of functions defined by $f_t(x) = t\ x^3 - \frac{1}{5}x^2 + \frac{1}{10}x$.

e) Calculate the coordinates, in terms of t, of all inflection points on the graph of f_t. [2]

f) This part-question involves the family of functions $f_t(x)$ as well as the parabola whose equation is given by $g(x) = 0.1 \cdot (x-10)^2 - 2$.

The distance d is defined as the distance between the vertex point of the parabola g and the point of inflection $W\left(\frac{1}{15\ t} \;\middle|\; \frac{1}{150\ t} - \frac{2}{3375\ t^2}\right)$ on the graph of f_t.

Find the value for t so that this distance d is a *minimum*. Hence, calculate this minimum distance. [3]

3. Calculus: Broken rational functions

Part 1

Consider the function f, defined by $f(x) = \frac{4x-4}{x^3}$.

a) Find any points of intersection between the graph of f and (i) the x-axis (ii) the y-axis. [1]

b) *Without the use of a calculator*, find and justify the equations of all asymptotes of the graph of f. [1]

c) Show, *without the use of a calculator*, that the graph of f has neither point symmetry about the origin nor line symmetry across the y-axis. [1]

d) *To obtain full marks for the next part-question, you must show a clear solution that does not use the calculator.*
Calculate the coordinates of all intersection points $S(x_S \mid y_S)$ between the graph of f and the graph of the function $g(x) = x^{-2}$. [1, 5]

e) Obtain an equation for the tangent line to the graph of f at the point where x = 3. [1, 5]

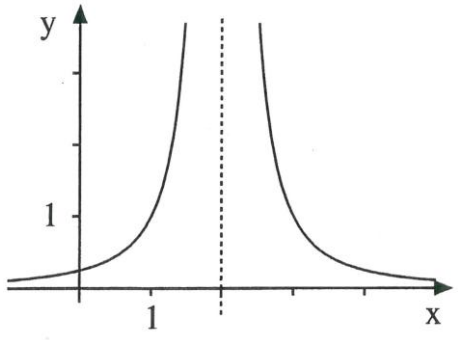
f) Show, *without use of a calculator*, that the function F defined by $F(x) = \frac{2}{x^2} - \frac{4}{x}$ is an Integral Function of f. [1]

g) The graph of f, the x-axis, and the vertical line with the equation x = k produce together a bounded region in the 1st quadrant whose area equals 0,5 units^2. Find, by calculation, the value of k. [1]

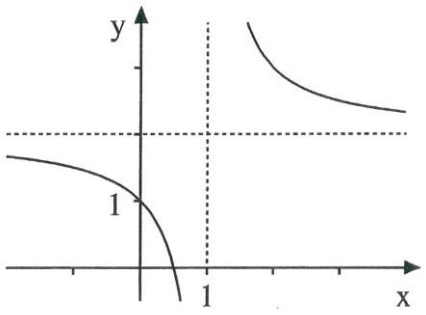
Part 2

a) Establish functions f_1 und f_2, respectively, which produce the graphs shown below.
The dotted lines on both graphs represent asymptotes. [2]

a_1)



a_2)



b) Give, where possible, the equation of a broken rational function for each of the following cases. If no such function exists for either or both of these cases, you should justify your decision with a brief and clear explanation.

Case b_1)
The graph of function f_3 has vertical asymptote $x = -4$. On both sides of this asymptote the function values approach minus infinity. The graph also has a horizontal asymptote with equation $y = -2$. [1]

Case b_2)
The graph of function f_4 has the vertical asymptote $x = 5$ and is symmetrical across the y-axis. [1]

4. Combinations and Probability

a) A company uses an Intelligence Test in order to decide which candidates it will employ. Four envelopes are provided: two containing a Mathematics question, one containing a Text Comprehension question and one containing a General Knowledge question.
A candidate who is invited to take the Intelligence Test estimates that her chance of success at the General Knowledge question is 80%. For the other two themes, she estimates her chance of success to be 70%. It may be assumed that these estimations of the candidate's ability at the different themes are realistic (i.e. accurate!). The candidate must choose an envelope at random and then provide an answer to the question which it contains.

a_1) What is the probability that the candidate chooses a question on General Knowledge and answers it correctly? [1]

a_2) What is the probability that this candidate does not pass the Intelligence Test ? [1, 5]

b) The company decides to hold an Open Day for students. It receives 45 applications (from 25 girls and 20 boys). However, only ten candidates will be invited, and these will complete their visit with a short interview. The supervisor of the selection process decides to choose these ten students at random.

b_1) In how many different ways can ten candidates be selected from these forty-five students? [1]

b_2) In fact, six girls and four boys are selected to attend the Open Day. In how many different ways was this possible? [1]

At the end of their visit, these six girls and four boys are interviewed one at a time,

b_3) In how many different arrangements may the students be interviewed? [1]

b_4) In how many different arrangements may the students be interviewed when it is agreed that all students of one gender will be interviewed first? [2]

c) The company is also looking for new employees in its Marketing Sector. From considerable experience, the company confidently estimates that 40% of applicants will have the necessary Qualifications and Professional Experience. On this occasion, six candidates are to be assessed.

c_1) Amongst these six candidates, what is the probability that *exactly* four of them will satisfy the requirements of the company? [1, 5]

c_2) Amongst these six candidates, what is the probability that *at least* four of them will satisfy the requirements of the company? [2]

c_3) The Selection Supervisor states : « If we have two candidates to asses, then the chance of at least one of them being suitable must be twice 40%. i.e. 80% ».
Is he correct ? [1]

5. Two independent questions

All numerical results to Question 5.1 should be rounded, where necessary, to two decimal places of accuracy.

5.1. The graph of function $f(x) = 2\sqrt{x} + 3$, $0 \le x \le 9$, together with the x-axis, the y-axis and the line $x = 9$ form a region called R_f. When this region is rotated 360 degrees around the x-axis, a solid, S_f in the shape of a flower vase (lying on its side) is formed. In each dimension, one unit equals one centimetre.

a) Produce a clear diagram, showing both the region R_f and the solid, S_f. [1,5]

b) Calculate the smallest and the largest diameter of the solid, S_f. [1]

c) Calculate, by hand, the volume of the solid, S_f. *For full marks, all stages of working must be shown.* [2,5]

d) Make the necessary calculations, to describe where a single cut through the solid should be made so that one of the two smaller solids produced will have a volume of exactly one litre. [2]

5.2. The line d: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$, the plane Π_2 : $2x - y - 2z + 5 = 0$ and the point A(1| –1| k) is given.

a) If it is known that the point A lies in the plane Π_2, what is the value of k? [1]

b) Show, with a calculation, that line d is parallel to the plane Π_2. [2]

c) Calculate the distance between the line d and the plane Π_2. [2]