gymnasium liestal

Maturitätsprüfungen 2010 – Mathematik schriftlich

Klassen: 4GL, 4LW (Bt, Fg)

Prüfungsdauer: 4h

Hilfsmittel: CAS-Taschenrechner (TI-89/Voyage 200, im Auslieferungszustand)
Taschenrechnerhandbuch (keine Notizen darin)
Formelsammlung Fundamentum (ohne Notizen darin)

Punkteverteilung:

1	2	3	4a	4b	5	Total
12	12	12	5	7	12	60

Question 1 - Analysis

Consider the family of curves $f_k(x) = \dfrac{x^4 + k}{x^2}$ where parameter $k \neq 0$.

(a) Does the graph of f_k possess any axis-symmetry and/or point-symmetry? (1 P.)

(b) Find the x-value of the vertical asymptote of the graph of the function. (1 P.)

(c) As $x \to \pm\infty$, to which function does $f_k(x)$ approach (asymptote)? (1 P.)

(d) For what parameter value(s) could this function possess any zero points? Give their x-axis intercepts, in terms of k. (1 P.)

(e) To obtain full marks for the following task, a **'by hand'** method should be shown. Calculate the coordinates of any stationary points on the curve of f_k, and state for which values of parameter k such points exist. (4 P.)

(f) Which value of k produces a point of inflection at $(3|y_W)$? Find y_W. (2 P.)

(g) Now consider the curves of $f_k(x)$ whose parameter values are $k = 1$ and $k = -1$. Calculate, for $x > 0$, the area of the region between these curves: from the x-value of the stationary point of one curve to the x-value of the inflection point of the other. (2 P.)

Question 2 - Vector Geometry

The points $A(-2|8|0), B(0|0|-2), C(1|2|0)$ and $D(0|6|1)$ are given.

(a) Show that the 4 points are the vertices of an isosceles trapezium. (2 P.)

(b) Give the Cartesian equation of the plane Π on which the trapezium $ABCD$ lies. (1.5 P.)

(c) Find the coordinates of the point M where the diagonals of the trapezium $ABCD$ intersect. (To achieve full marks for part (c) a 'by hand' method must be shown.) (2 P.)

(d) Find the interior angles of the trapezium $ABCD$. (1.5 P.)

(e) Find the area of the trapezium $ABCD$. (2 P.)

The trapezium $ABCD$ and a point S form a pyramid. S lies on a straight line g which is normal to the plane of the trapezium and passes through a point $P(0|4|0)$ which lies on plane Π. The height of the pyramid is 15 and point S and the origin lie on opposite sides of plane Π.

(f) Calculate the coordinates of point S the apex of the pyramid.
(If you are unable to find an equation for part (b) use this equation for plane $\Pi : -2x - y + 2z + 4 = 0$) (3 P.)

Question 3 - Theory of Probability

In the recent Swine Flu (H1N1 virus) epidemic, a test for the H1N1 virus was carried out on a sample of 100 people, 20 of whom had the virus. Some of the results are shown in this table:

	People with the virus	People without the virus	Total
Test result positive	19	A	B
Test result negative	C	78	D
Total	20	E	100

(a) Use the information given above to find values for A, B, C, D and E. (1 P.)

Using the table or otherwise, find the probability that a randomly selected individual in the sample:

(b) gets a positive result, (1 P.)

(c) does not have the virus and is tested positive, (1 P.)

(d) gets a negative result given that s/he has the virus. (1 P.)

Of the 80 people in the above sample without the virus, 50 were male. Researchers wanted a group of 30 people from the 80 without the virus on whom to conduct further experiments. In how many different ways could a group of 30 be selected from the 80 if:

(e) There were no restrictions, (1 P.)

(f) Exactly half the group of 30 must be females? (1.5 P.)

(g) Say a group consisting of 15 males and 15 females had been selected which the researchers wanted to subdivide to conduct three types of experiment. In how many ways could these three different sub-groups be formed from the 30 selected people, if each sub-group had 5 males and 5 females? (2 P.)

In the general population it was estimated that the probability of an individual having the H1N1 virus was around $0,01\%$. Assuming that this probability is correct and that the virus is randomly distributed in the population,

(h) Find the probability that in a class of 24 students exactly two students have the virus, (1.5 P.)

(i) With how many people at least would one need to be in contact, for the probability of meeting at least one with the Swine Flu virus to be greater than 99%? (2 P.)

Question 4a - Optimisation problem

A rectangle with side lengths x and y is given. When the rectangle is rotated 360° about its side length y, a cylinder is created.

(a) Produce a clear sketch of this situation. (1 P.)

(b) The perimeter of the rectangle is 18 cm. Calculate the volume, $V(x)$, of the cylinder, in terms of x. (1 P.)

(c) State a realistic domain for $V(x)$. (1 P.)

(d) To obtain full marks for the following task, a **'by hand'** method should be shown. Calculate the values for x and y which produce a cylinder with the largest volume. (2 P.)

(If you have been unable to obtain an expression for $V(x)$ in part (b), you should answer part (d) using $V(x) = \pi \cdot (12x^2 - x^3)$.)

Question 4b - Exponential Growth

Adenocarcinoma is a rapidly growing form of tumour. If N is the number of tumour cells and t is the time in months the equation $N(t) = 100 \cdot 2^{t/3}$ describes how the number of tumour cells increases with time.

(a) How many tumour cells are present at $t = 0$? (0.5 P.)

(b) How long does it take for the number of tumour cells to double? (1 P.)

(c) An approximately spherically-shaped tumour can be detected by doctors when it is at least 1 cm in diameter, which is about 10^9 tumour cells. At what time t can a tumour first be detected? (0.5 P.)

(d) Exactly seven years after t=0, Adenocarcinoma is detected in a person, what is the radius of the sphere of tumour cells? (1.5 P.)

(e) The treatment of the person in part (d) starts exactly one month after the tumour is detected, what is the rate of growth of the tumour (number of cells per month) when the treatment starts? (1 P.)

(f) This treatment destroys 99.9% of the tumour cells. If no further treatment were given, how long would it take for the remaining cells to increase to the point where the tumour can be detected? (To achieve full marks for part (f) a **'by hand'** method must be shown.) (2.5 P.)